

Pairing and symmetry energy in $N \simeq Z$ nuclei

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Abstract

It is shown that in $N \simeq Z$ nuclei the effects of isovector pairing and the symmetry energy are intimately related. In particular, in the odd-odd $N = Z$ nuclei these two effect are essentially equal in magnitude, causing near degeneracy of the lowest $T = 1$ and $T = 0$ states and appearance of the $T = 1$ ground state isospin in many such nuclei. Following the earlier work of Jänecke, it is shown that the global symmetry energy fit can be reproduced using just the excitation energies of the lowest $T = 3/2$ states in $N = Z + 1$ nuclei. Similarly, the global pairing gap fit can be related to the excitation energy of the $T = |N - Z|/2 + 1$ states in $N \simeq Z$ even A nuclei.

21.10-k, 21.10.Dr, 21.60.-n

The treatment of pairing correlations in proton rich $N \simeq Z$ nuclei, where the neutron-proton (np) pairing is expected to play an important role, became a popular subject recently [1–7]. Arguments based on isospin symmetry suggest that the isovector np , $J^\pi = 0^+$ pairing should be considered on the same footing as the conventional like-particle nn and pp , $J^\pi = 0^+$ pairing correlations. In addition, neutrons and protons can also interact via an isoscalar force; this new mode of pairing with $J \neq 0$ has been also recently extensively discussed [8,9].

The obvious experimental signature of pairing correlations is the extra binding energy of the even-even nuclei relative to their odd- A neighbors. (This feature is present even in the more general situation as shown in Ref. [10].) Moreover, in stable nuclei with $N > Z$, odd-odd nuclei are even less bound than their odd- A counterparts. Now, if pairing correlations indeed have an isovector character, then the odd-odd $N = Z$ nuclei should be as fully paired as the even-even $N = Z$ nuclei are and one could therefore expect that their masses form a single smooth line as a function of the mass number A . However, as shown in Fig. 1, the experiment does not support this assumption. In fact, the opposite is true: as one can see the odd-odd nuclei are systematically less bound than the even-even ones. Does it mean that the idea of isovector pairing is incorrect and the np pairing is absent? The answer is no. In this paper I explain this seemingly paradoxical situation, and show its relation to the interplay between the pairing and symmetry energies. This is a phenomenological analysis based on the experimental nuclear masses and excitation energies. No attempt is made to relate the facts to the underlying many-body theory. Instead, the smooth trends are revealed and categorized. I concentrate on the effects associated with the isovector pairing; it is assumed that the isoscalar pairing is included as a component of the nuclear symmetry energy.

The considerations below follow the much earlier work of Jänecke [11] which, in turn, has been apparently in part inspired by an even older work of Baz and Smorodinsky [12]. In Ref. [11] the empirical values of the symmetry and pairing energies were deduced from the excitation energies of the analog states. These then could be used in order to predict the masses of unknown nuclei. Our motivation here is different. Nevertheless, there is a close connection with the work of Jänecke [11], as shown throughout the text below.

The clue to the paradox in Fig.1 is the isospin. All even-even $N = Z$ nuclei have $T = 0$ ground states, while the odd-odd $N = Z$ nuclei have either $T = 1$ ground state or the $T = 1$ state is one of the low-lying states in the spectrum. It is well known [13] that nuclear binding depends on the smooth volume, surface, and Coulomb energies and, in addition, on the symmetry energy which reduces the binding energy of the $T = 1$ states with respect to the $T = 0$ states, explaining the trend in Fig.1. Thus, in the $N \simeq Z$ nuclei one must not treat the pairing alone; the effect of the symmetry energy should be included as well.

Consider the ground state of a nucleus with $A = N + Z$ nucleons, with A even. This state can be a condensate of only $J^\pi = 0^+$ isovector pairs if $T = |N - Z|/2$ for even-even nuclei, and if $T = |N - Z|/2 + 1$ for the odd-odd nuclei. In other words, states with $T = |N - Z|/2 + 1$ in even-even nuclei must contain at least one broken pair, and similarly states with $T = |N - Z|/2$ in odd-odd nuclei must also contain at least one broken pair. In contrast, in the odd- A nuclei one can have states with both $T = |N - Z|/2$ and $T = |N - Z|/2 + 1$ all having just one unpaired nucleon. Based on these considerations one can understand the trends shown in Fig. 2. The quantities $\Delta_{T',T}$ plotted in Fig. 2 are the excitation energies of the lowest states of isospin T' with respect to the states with isospin

T . Analogous figures can be found in Refs. [11,12].

Let us discuss first the middle panel b) in Fig. 2 for the odd- A $N = Z + 1$ nuclei. Disregarding the relatively small shell effects, the excitation energies of the lowest $T = 3/2$ state form a smooth monotonic curve. Indeed since, as I argued above, pairing should be the same for both isospins, the excitation energy should be simply related to the symmetry energy. In fact, the dashed line in panel b), based on the symmetry energy formula fitted in Ref. [14],

$$E_{sym} = \frac{T(T+1)}{A} \left[134.4 - 203.6 \frac{1}{A^{1/3}} \right] \text{ (MeV) ,} \quad (1)$$

approximates the data very well. Note that this formula results from the fit to masses of 1751 nuclei (together with the other parameters of the set 6p) in Ref. [14]. Here I confirm its validity using just few excitation energies, and in fact it should be possible to fit it independently from the data in the panel b) alone. In particular, the sign and magnitude of the correction (surface) term is verified.

However, the even- A nuclei in panels a) and c) are quite different. There, the excitation energies form *two* curves as already noted in Refs. [11,12]. In the upper one for the even-even nuclei the symmetry energy and pairing add, and in the lower one for the odd-odd nuclei the excitation energy is the difference between the symmetry energy effect and pairing, i.e. they act against each other. That this is the correct interpretation of the data is verified by the fact that the long dashed lines in panels a) and c), based on the symmetry energy, Eq. (1), bisect the two curves. (These lines are not exactly in the middle of the two lines, in particular in c). I will comment on the possible reason for this later.) Thus, while the average of the even-even and odd-odd curves in Fig. 2 represents the symmetry energy, their difference represents twice the energy needed to break an isovector pair. Fig. 2 is based on excitation energies as listed in Table of Isotopes [15]. In few cases, however, the isospin assignment is not available. In those cases I used the mass of the corresponding analog nucleus, corrected for the neutron-proton mass difference and the Coulomb energy also based on Ref. [14]

$$E_{Coul} = -0.699 \frac{Z(Z-1)}{A^{1/3}} \left[1 - \frac{0.76}{[Z(Z-1)]^{1/3}} \right] \text{ (MeV) .} \quad (2)$$

In cases where it can be checked this formula gives quite good agreement with the excitation energies displayed in Fig. 2. However, the Coulomb energy is otherwise irrelevant for the further discussion.

Several features of Fig. 2 deserve special comment. First, one can see that in the $N = Z$ odd-odd nuclei the lowest $T = 1$ and $T = 0$ states are almost degenerate. There is very little difference in this respect between the sd shell nuclei ($A < 40$), where $T = 0$ is the ground state, and the heavier pf shell nuclei where the $T = 1$ is usually the ground state. The small difference seen in Fig. 2 can be obviously correlated with the curvature of the symmetry energy line; it has apparently little to do with the strength of the isovector and isoscalar pairing. This degeneracy of the $T = 0$ and 1 states means that, remarkably, in these nuclei the symmetry and pairing energies are equal in magnitude, i.e. *almost exactly cancel* each other. Based on the figure one would expect that the $T = 1$ and $T = 0$ states will remain close to each other also in the heavier $N = Z$ odd-odd nuclei.

One can understand also that the odd-odd $N = Z$ nuclei are the only ones known that violate the rule that the ground state isospin is $T = |N - Z|/2$. In essentially all other nuclei the symmetry energy is stronger than pairing, such as in c) and even more so in nuclei with larger $|N - Z|$ and/or ground state isospin. In Ref. [11] the symmetry and pairing energies were fitted from the quantities $\Delta_{T',T}$, with the shell effects included. Based on these fits, Jänecke [11] predicted that the isospin “inversion” should also occur in other odd-odd very proton rich nuclei, e.g. for $T_z = \pm 1$ for $108 \leq A \leq 124$ as well as for $A \geq 192$, and for $T_z = \pm 2$ for $A \geq 290$. Most of such nuclei are, however, beyond the proton drip line.

Further, it is clear that the symmetry energy should depend on the isospin T and not just on the square of the neutron excess $(N - Z)^2$ as in the usual liquid drop formula. (In most heavier nuclei, however, where the liquid drop formula is used, $T = |N - Z|/2$ is much larger than unity and the difference between $T(T + 1)$ and $(N - Z)^2/4$ plays very little role.) The parametrization in Eq. (1), containing $T(T + 1)$, is motivated by the charge independence of the nuclear force. But as a phenomenological parametrization the isospin dependence $T(T + a)$ with $a \neq 1$ is also possible. In fact, the empirical fits in Ref. [11] show a clear preference for $a = 1$; the fits with either $a = 0$ as in the usual liquid drop formula, or with $a = 4$ as in the Wigner SU(4) symmetry are disfavored. Similar conclusion, namely that $a \simeq 1$, has been reached in Ref. [16].

The presence of a term linear in $T = |N - Z|/2$ has a special relevance for the so-called Wigner energy¹. The term, first introduced by Wigner [17], is used for the additional binding energy in the semi-empirical mass formulae,

$$E_W = W(A)|N - Z| + d(A)\pi_{np}\delta_{NZ} , \quad (3)$$

where $W(A)$ is a smooth function of A describing the magnitude of the effect. (The quantity $\pi_{np} = 1$ for odd-odd nuclei and vanishes otherwise. The $d(A)$ term, relevant only in the odd-odd $N = Z$ nuclei is not discussed further here.) The empirical fit to the first term in Eq. (3) gives $E_W \simeq 47|N - Z|/A$ MeV [18]. The symmetry energy formula, Eq.(1) contains a term (for $A \simeq 40$) $37|N - Z|/A$ MeV not very far from the empirical Wigner energy value. This finding is in agreement with Ref. [7] where it was shown that the experimental magnitude of the Wigner term is reduced substantially if one uses $T(T + 1)$, instead of the more common $(N - Z)^2$ for the symmetry energy.

While reducing the need for the extra Wigner energy term, Eq. (1) does not eliminate it completely. This could be achieved, perhaps, by using the form $T(T + a)$, $a > 1$ for the symmetry energy. Such a choice would also move the middle curve in panel c) of Fig. 2 towards the average of the curves for even-even and odd-odd nuclei. Thus, there is an indication that the “best” semi-empirical symmetry energy formula would have $a > 1$. No attempt has been made to do such a fit here; a more physical approach would relate the Wigner energy to the various components of the neutron-proton force such as in Ref. [18].

What is then the magnitude of the pairing gap in the $N \simeq Z$ nuclei? The usual definition [13] relates the gap to the binding energy difference of the given even- A nucleus to its odd- A neighbors eliminating other smooth trends. For example, for the neutron gap one is supposed to use

¹Prof. A. Zuker stressed this point in a private communication to the author

$$\Delta_{nn}(N, Z) = \frac{1}{4} [B(N-2, Z) - 3B(N-1, Z) + 3B(N, Z) - B(N+1, Z)] . \quad (4)$$

However, in $N \simeq Z$ nuclei such definition is not really applicable, since not only the number of the nn pairs changes, but also the number of np pairs.

We can use, however, the excitation energy of the lowest $T_> = |N - Z|/2 + 1$ state with respect to the lowest $T_< = |N - Z|/2$ one, as shown in Fig. 2, for this purpose. Since the lines are there just to guide the eye, one must use some form of interpolation, however. In Fig. 3, I use the simplest averaging in order to obtain the gap for an even-even nucleus,

$$\begin{aligned} \Delta_{ee}(N, Z) = & \frac{1}{4} \{ E_{ee}(N, Z, T_>) - E_{ee}(N, Z, T_<) \\ & - \frac{1}{2} [E_{oo}(N-1, Z-1, T_>) + E_{oo}(N+1, Z+1, T_>) \\ & - E_{oo}(N-1, Z-1, T_<) - E_{oo}(N+1, Z+1, T_<)] \} . \end{aligned} \quad (5)$$

An obvious modification is used for the odd-odd nuclei.

The results shown in Fig. 3 show, first of all, that the pairing gaps for a given $N - Z$ form a more or less continuous curves, with however clearly visible shell effects. The gaps defined using Eq. (5) do not vanish at magic numbers, showing that the odd-even staggering is indeed a more general feature of nuclear spectra, as suggested in Ref. [10]. The smooth trends can be fitted as $6.24/A^{1/3}$ MeV for the $N = Z$ nuclei and $5.39/A^{1/3}$ MeV for the $N = Z + 2$ nuclei. (The curve for the $N = Z + 4$ nuclei, not shown, is very close to the one for $N = Z + 2$.) In Ref. [14] the general pairing gap was fitted as $5.18/A^{1/3}$ MeV, quite close to our fit for $N > Z$. The larger gap for $N = Z$ reflects the gain in pairing due to stronger np correlations.

In conclusion, following Ref. [11] we have seen that in the $N \simeq Z$ nuclei the effects of pairing and symmetry energy are closely related. In particular, in the odd-odd $N = Z$ these two effect are essentially equal in magnitude, and their cancellation causes the near degeneracy of the lowest $T = 1$ and $T = 0$ states. We have also shown that the symmetry energy extracted from the excitation energies of the $T_>$ states agrees with the global fit. Moreover, the considerations presented here show that the proper parametrization of the symmetry energy must be a quadratic function of the isospin T of the form $T(T + a)$ (and not $(N - Z)^2$). Based on isospin symmetry, one would naturally choose $a = 1$, but the purely empirical value, which would at the same time explain most of the Wigner energy, favors somewhat larger value of a . Finally, the pairing gaps extracted from these data by the procedure in Eq. (5) agree with the usual definition and global fit for the $N > Z$ nuclei. We find evidence for about 20% increased gaps in the $N = Z$ nuclei.

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FIGURES

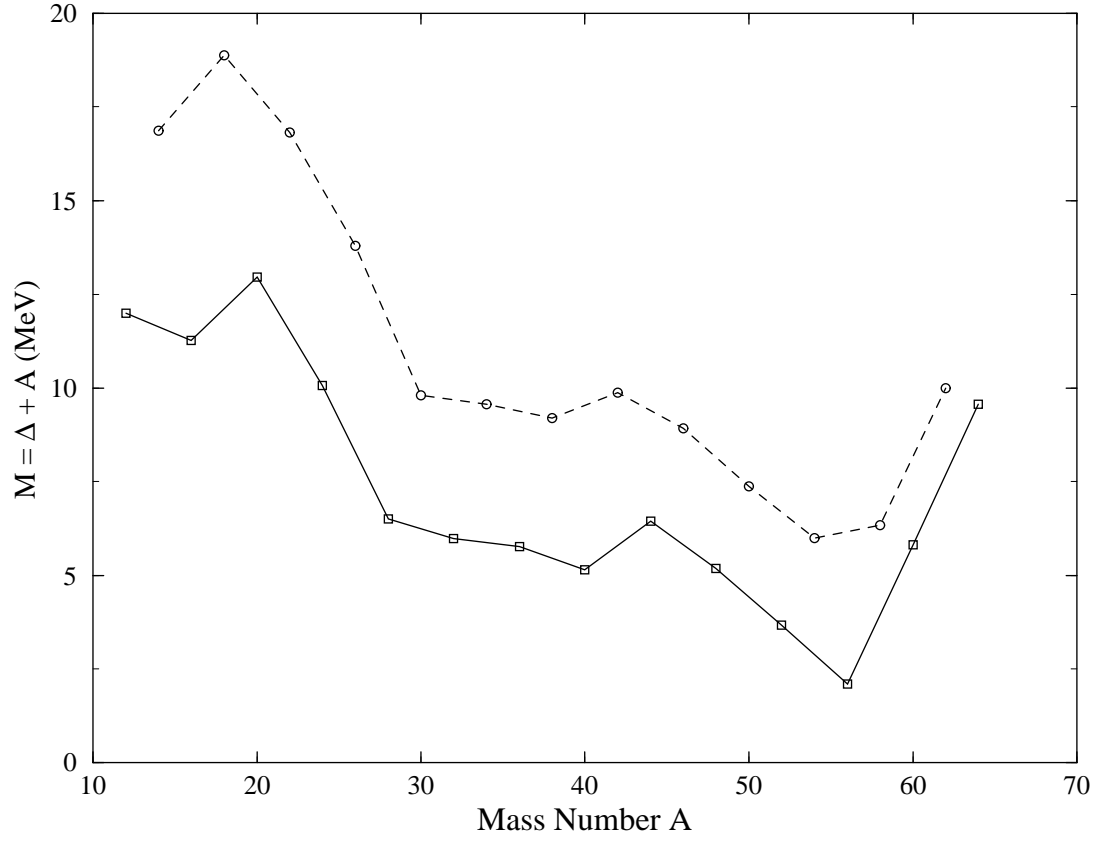


FIG. 1. Mass excess of the $N = Z$ nuclei. The dashed line with circles connects the masses of the odd-odd nuclei and the full line with squares the even-even nuclei. For easier viewing, the $\Delta + A$ instead of Δ is plotted versus A .

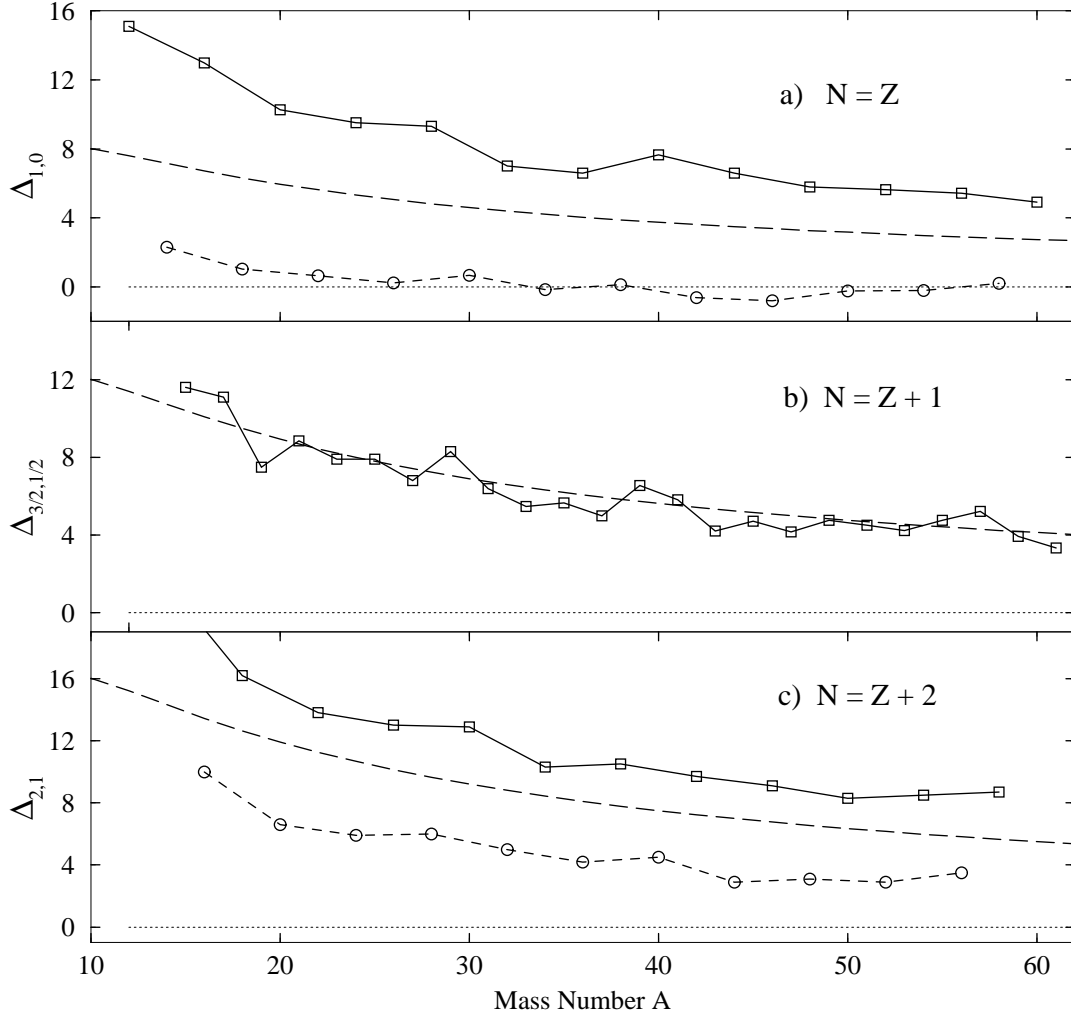


FIG. 2. Experimental excitation energies $\Delta_{T',T}$ (in MeV) of the lowest $T' = |N - Z|/2 + 1$ state with respect to the lowest $T = |N - Z|/2$ state. Panel a) is for the $N = Z$ nuclei, b) is for the $N = Z + 1$ nuclei, and c) for the $N = Z + 2$ nuclei. In panels a) and c) the squares connected by full lines to lead the eye are for the even-even nuclei, and the circles connected by short dashed lines are for the odd-odd nuclei. In all three panels the long dashed line is the symmetry energy difference defined in Eq. (1). The thin dotted lines indicate zero excitation energy.

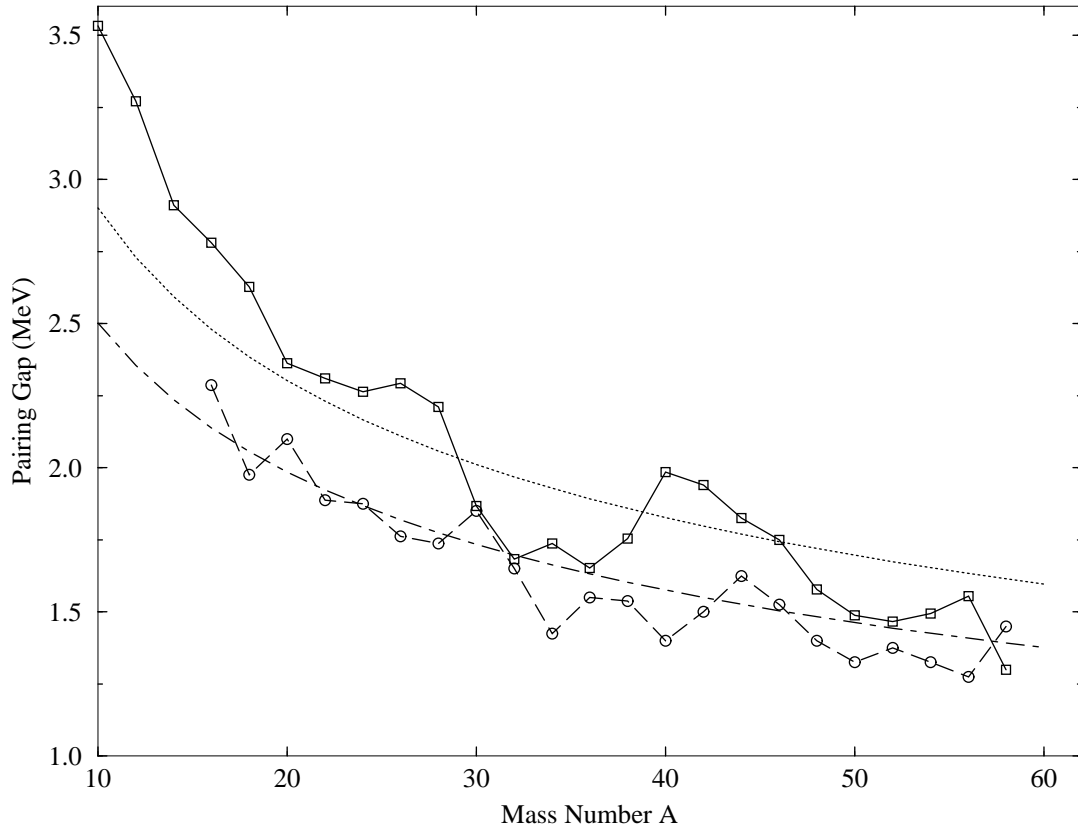


FIG. 3. Pairing gaps for the $N = Z$ nuclei (squares connected by the solid line) and for the $N = Z + 2$ nuclei (circles connected by the long dashed line). The smooth fits are $6.24/A^{1/3}$ (dotted line for $N = Z$) and $5.39/A^{1/3}$ (dot-and-dashed line for $N = Z + 2$). All gaps are in MeV and were extracted by the method described in the text.